Comments and Corrections

Comments on "Effect of Vergence Adaptation on Convergence Accommodation: Model Simulations"

George K. Hung

I. INTRODUCTION

A. *Overview*

In [1], Sreenivasan *et al.* claimed that their experimental results matched more closely to Schor's [2] model than that of Hung's model [3] and, thus, validated Schor's model (see Fig. 1).

Sreenivasan *et al.* [1] made the assumption that convergence accommodation output, which is a model internal parameter, can be obtained as the difference between the accommodative response under the accommodation closed-loop (CL) and vergence CL condition and that under the accommodation CL and vergence open-loop (OL) condition. This is fundamentally flawed because in a feedback control system, the component outputs are not additive if the model configuration is changed due to a change in the stimulus condition. (They are additive only if the stimulus condition remains the same; then the principle of superposition holds). Moreover, the behavior of a model in the CL condition is very different from that in the OL condition. Therefore, adding or subtracting the effects in the CL and OL conditions to obtain an internal parameter is incorrect. Moreover, since the main premise of their study is based on this false assumption, their conclusion is invalid.

B. *Specific Problems With [1, Fig. 8(b)]*

There are two ordinate scales that are combined in [1, Fig. 8(b)], the lower half of the ordinate scale ranges from 0 to 0.9, while the upper half ranges from 1 to 6. The starting points for the model simulations are at about 3.0, whereas the experimental data [4] start at about 0.5. This exaggeration of scales inappropriately suggests a dynamic correlation between two of the curves. On a normal scale from 0 to 6, the experimental data would be close to a horizontal line near the zero level. Hence, the transient component of the experimental data is almost negligible in such a standard-size plot. Such a standard-size plot, however, can provide a comparison of the responses at the steady-state levels, i.e., the later horizontal levels well after the adaptation has been completed. Since all models should exhibit the correct steady-state response, a direct comparison can be made between the responses shown in Fig. 1 and the static model response. Indeed, as I explain in detail in the following section, quantitative analysis of the steady-state model responses shows not only that the authors' assumption about subtracting CL and OL responses to obtain an internal parameter (i.e., convergence-accommodation) is incorrect, but that, in fact, the expected value based on such a subtraction (a small value, which is far from the larger calculated convergence accommodation value) is consistent with the experimentally derived subtracted value. To put it another way, the authors' experimental data in fact verifies the steady-state model configuration in showing that a subtraction of accommodative outputs under CL and OL conditions will result in a small value, which is consistent with the value derived analytically (but this value is much smaller than the calculated convergence accommodation value).

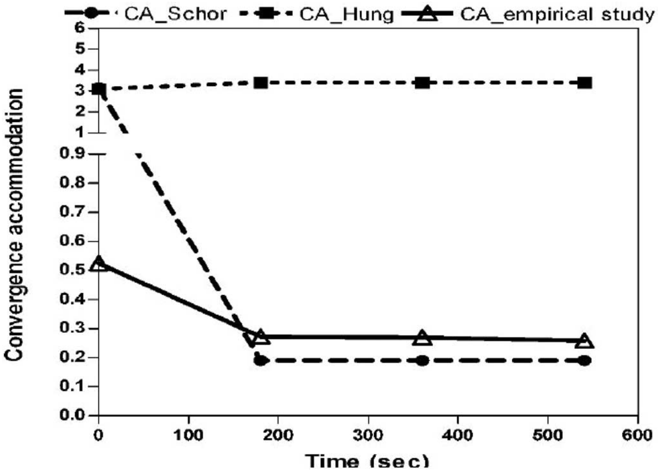

Fig. 1. Model and experimental convergence accommodation responses. Reproduced from the Sreenivasan *et al.* [1, Fig. 8(b)].

There is a further consequence in their steady-state results. The small steady-state value of Schor's model is much smaller than the convergence accommodation value derived analytically, thus showing some kind of error in Schor's model. This conclusion is the opposite of those by the authors.

II. QUANTITATIVE ANALYSIS OF THE MODEL

The following analysis is provided to show quantitatively the fundamental problem with the authors' assumption of additivity of components under CL and OL conditions to obtain the output of an internal parameter.

A. *Adaptation Model of Accommodation and Vergence*

Hung's [3] adaptation model of accommodation and vergence is shown in Fig. 2. As discussed earlier regarding [1, Fig. 8(b)], the portion of the authors' data that can provide any basis of comparison is at the steady-state level, which is well after adaptation has taken place. Such steady-state behavior can be analyzed by means of a static model. The static model can be derived from the adaptation model by removing the adaptation components. Also, the parameters depth of focus, Panum's fusional area, tonic accommodation, and tonic vergence are excluded because they would have only slightly modified the model responses, while adding greater complexity to the model equations.

B. *Static Model of Accommodation and Vergence*

After the transients have died out, the interactive systems can be represented by a static model of accommodation and vergence (see Fig. 3). In [1, Fig. 8(b)], the static condition is seen for over two-third of the time scale, and it is over this interval that the difference between the two models is manifest and would serve as the basis for the comparison made by the authors. Thus, quantitative analysis of the static model provides a definitive means for comparing the two models.

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G. K. Hung is with the Biomedical Engineering Department, Rutgers University, Piscataway, NJ 08854 USA (e-mail: shoane@rci.rutgers.edu).

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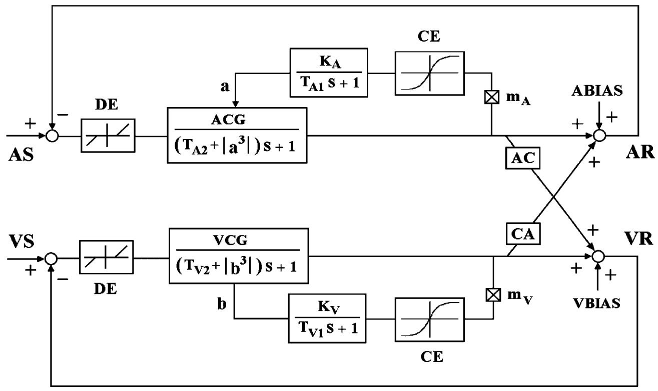

Fig. 2. Adaptation model of combined accommodation and vergence system. The deadspace element DE for accommodation represents the depth of focus, and DE for vergence represents Panum's fusional area. ABIAS represents tonic accommodation and VBIAS represents tonic vergence. The time constants T_{A_2} and T_{V_2} of the accommodative and vergence controllers, respectively, are modified by their adaptive components. In each feedback loop, the adaptive component consists of a constant gain (m_A or m_V) element, a compression element (CE), and a first-order dynamic controller. From Hung [3].

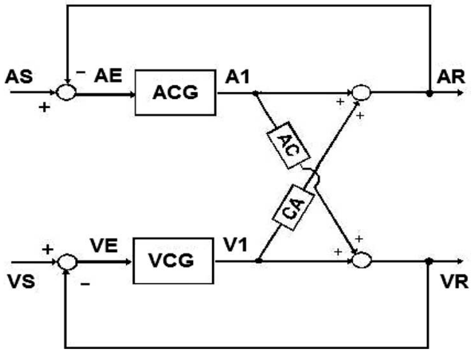

Fig. 3. Simplified static accommodation and vergence model.

For the aforementioned model (see Fig. 3), it can be shown [5] that the accommodative response under CL accommodation and vergence is given by (1) at the bottom of the page. Also, the accommodative response with CL accommodation and OL vergence (the vergence loop is opened by breaking the connection at VE in Fig. 3) is given by (2) at the bottom of the page. Moreover, the convergence accommodation output (i.e., the value after the CA crosslink in Fig. 3) with CL accommodation and vergence is given by (3) at the bottom of the page. Clearly, the difference between the equations for $\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ and $\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{OL}}}$ [or (1) minus (2)] is not equal to the equation for $\mathrm{CA\_OUTPUT}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ [see (3)]. Thus, Sreenivasan *et al.*'s assumption of an additive effect of CL and OL responses is incorrect.

TABLE I
STATIC MODEL CALCULATED RESPONSES FOR THE SAME STEADY-STATE STIMULUS CONDITIONS AS THOSE IN SREENIVASAN *et al.* [1]

Condition	AS	VS	AR	VR	CA_OUTPUT $A_{CL}V_{CL}$
$A_{CL}V_{CL}$	1	3	1.30	2.98	**3.88**
$A_{CL}V_{OL}$	1	----	0.90	0.76	0
Difference	---	----	**0.40**	----	----

This can be further illustrated by substituting quantitative values into the model, with ACG = 8.53, VCG = 293, AC = 0.85, and CA = 0.75 (from Hung and Semmlow [6]; these model parameters values were also used in Sreenivasan *et al.* [1]). The model responses are shown in Table I.

The difference between $\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ and $\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{OL}}}$ equals 0.40, which is an order of magnitude smaller than $\mathrm{CA\_OUTPUT}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ (equal to 3.88), see (1)–(3).

Overall, the aforementioned analysis showed that the difference between the equations for $\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ and $\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{OL}}}$ [(1) minus (2)] is *not equal* to the equation for $\mathrm{CA\_OUTPUT}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ [see (3)], which is the parameter the authors thought they were estimating by such a subtraction. In fact, the internal parameter $\mathrm{CA\_OUTPUT}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ [see (3)] was calculated to be 3.88 (see Table I), which is close to the ∼3.5 for Hung's model convergence-accommodation output obtained by the authors' simulation in [1, Fig. 8(b)]. Also, the authors' experimental data for the difference between $\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ and $\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{OL}}}$ is ∼0.25 [see Fig. 8(b)], which is close to the Hung model difference value of 0.40 [see Table I; (1) minus (2)]. This demonstrates a consistency between model and experimental data, but again this value is not equal to the internal parameter that they sought, or $\mathrm{CA\_OUTPUT}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ [see (3)]. Moreover, Schor's value of ∼0.19 (see [1, Fig. 8(b)]), which is supposed to represent his model's internal convergence-accommodation output, is a small value (that is coincidentally) close to that based on the difference between $\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ and $\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{OL}}}$, (equal to ∼0.25 for the authors' data (see [1, Fig. 8(b)]), and 0.40 for the Hung model analytical-derived value (see Table I), but is far different from the 3.88 value expected for $\mathrm{CA\_OUTPUT}_{A_{\mathrm{CL}}V_{\mathrm{CL}}}$ [see (3)]. This demonstrates that Schor's model steady-state response is incorrect.

III. SUMMARY

Sreenivasan *et al.* [1] have used an inappropriate measure to estimate convergence accommodation under CL accommodation and vergence, which they then used to compare the two adaptation models. In [1, Fig. 8(b)], they used a dual-scale plot to exaggerate the small change in experimental dynamics, but such a plot could only appropriately provide a comparison of steady-state level responses. Quantitative analysis

$$\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{CL}}} = \frac{\mathrm{ACG}\times[(1+\mathrm{VCG})\times\mathrm{AS}-\mathrm{VCG}\times\mathrm{CA}\times\mathrm{VS}]+\mathrm{VCG}\times\mathrm{CA}\times[(1+\mathrm{ACG})\times\mathrm{VS}-\mathrm{ACG}\times\mathrm{AC}\times\mathrm{AS}]}{(1+\mathrm{ACG})\times(1+\mathrm{VCG})-\mathrm{ACG}\times\mathrm{VCG}\times\mathrm{AC}\times\mathrm{CA}} \tag{1}$$

$$\mathrm{AR}_{A_{\mathrm{CL}}V_{\mathrm{OL}}} = \frac{\mathrm{ACG}\times\mathrm{AS}}{1+\mathrm{ACG}} \tag{2}$$

$$\mathrm{CA\_OUTPUT}_{A_{\mathrm{CL}}V_{\mathrm{CL}}} = \frac{\mathrm{VCG}\times\mathrm{CA}\times[(1+\mathrm{ACG})\times\mathrm{VS}-\mathrm{ACG}\times\mathrm{AC}\times\mathrm{AS}]}{(1+\mathrm{ACG})\times(1+\mathrm{VCG})-\mathrm{ACG}\times\mathrm{VCG}\times\mathrm{AC}\times\mathrm{CA}} \tag{3}$$

of the static model of accommodation and vergence showed that subtraction of CL and OL responses resulted in a small numerical value, which is close to the authors' experimentally derived value. The authors had assumed erroneously that such a difference would represent the convergence accommodation response. But their value is very different from the large numerical value for convergence accommodation calculated analytically. In the model simulations, the large difference in steady-state convergence accommodation levels for the two models provides a means to assess their appropriateness. Schor's model response exhibited a very small value. On the other hand, Hung's model response exhibited a large value, which is close to the level calculated analytically.

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Authors Reply

Vidhyapriya Sreenivasan*, William R. Bobier, Elizabeth Irving,
and Vasudevan Lakshminarayanan

We appreciate the opportunity to respond to Dr. Hung's comments [1] regarding our study [2]. We emphasize that the primary purpose of the study was to determine if the two most commonly cited dynamic models of accommodation and vergence [3], [4] predicted two empirical findings taken from our investigation of near addition lenses [5]. First, we examined the change in vergence (phoria) that occurred upon lens addition and then concurrent changes in (con) vergence-driven accommodation (CA). We examined the vergence response (phoria) and CA (defined as the difference between binocular–monocular accommodations) over a 20-min period. Changes in tonic levels of vergence were measured by plotting the changes in near phoria. Both models correctly predicted the pattern of change in tonic vergence found in our empirical measures. This change is termed vergence adaptation and represents the replacement of fast (reflex) vergence by that of a slow (tonic) vergence. However, the reduction in CA that occurred concurrent with vergence adaptation was only predicted by the model described by Schor [3]. In [2], we *compared the (relative) pattern of change* in CA between experimental data and model simulations and *did NOT compare* the absolute magnitude obtained from the models. This was similar to how we analyzed vergence adaptation. We have specifically stated this approach in the introduction section of our paper (last para, page 2390). "We used the results of this empirical study to compare the *relative pattern of changes* observed by simulating the two models of adaptation."

We understand that a primary concern of Dr. Hung appears to be our estimation of empirical CA. No objection was provided in regards to vergence adaptation. However, Dr. Hung chose to formulate his objection to CA based upon quantitative analysis, using a static model. It must be understood that static models do not include vergence adaptation, and these models are inappropriate for these data since vergence adaptation has occurred. Dr. Hung tries to compare the absolute response of CA at a steady-state level, which was NOT the aim of the study. We deal with each specific issue shortly:

Estimation of Empirical CA (Binocular–Monocular Accommodation)

It is generally held (see Heath [6] for a review) that the total accommodative response to a given target is composed of contributions from blur, vergence, proximal, and tonic components of accommodation. In our empirical study that used +2D lenses, we have ensured that the outputs from proximal, tonic and blur accommodation are constant, and thus any changes to binocular accommodation can be attributed to the disparity-induced component (CA crosslink) of accommodation. The proximal cue was held constant by taking measurements at the same testing distance, and constant tonic accommodation was ascertained by confirming no adaptation of tonic accommodation [2, p. 2392, last para of empirical results], and a stable blur accommodation was determined by the steady monocular response [2, Fig. 4]. Consequently, the reduction of binocular accommodation [2, Fig. 4], concurrent with the reduction of exophoria [2, Fig. 3] does indeed represent the reduction of CA crosslink with vergence adaptation.

Furthermore, to confirm our statement (i.e., CA reduces with vergence adaptation), we performed an additional experiment, using wedge prisms with the same stimulus conditions as our paper [2] (VS = 1; AS = 3) but instead of using the difference between binocular and monocular accommodation, we directly measured changes to CA (open-loop accommodation and closed-loop vergence, $\mathrm{AR}_{\mathrm{OL}}$ $\mathrm{VR}_{\mathrm{CL}}$) with prolonged near task (unpublished data). The resulting patterns of CA were then compared to the results of model simulations, also obtained under $\mathrm{AR}_{\mathrm{OL}}$ $\mathrm{VR}_{\mathrm{CL}}$ condition. Direct measures of CA also show a pattern of decrease in the crosslink activity with vergence adaptation, similar to our paper [2] that estimated binocular–monocular accommodation. The pattern of decline in CA following vergence adaptation is well described by Schor's model [3]. Hung's model on the other hand [4] showed a CA output that remained unchanged with vergence adaptation, and this constancy of CA has been acknowledged by the author in his paper [4].

The experimental results of reduced CA crosslink activity with vergence adaptation are analogous to a study that measured the effect of positive fusional vergence training on vergence adaptation and CA in 11 adult subjects [7]. CA was elicited by the use of wedge prisms. The authors report a consistent pattern of reduction in CA with vergence

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*V. Sreenivasan is with the School of Optometry, University of Waterloo, Waterloo, ON N2L3G1, Canada (e-mail: vsreeniv@uwaterloo.ca).

W. R. Bobier, E. Irving, and V. Lakshminarayanan are with the School of Optometry, University of Waterloo, Waterloo, ON N2L3G1, Canada.

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